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Nearest-neighbor-spacing distribution of a system with many degrees of freedom, some regular and some chaotic

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We consider a quantum system with a Hamiltonian expressed as a sum of two terms. The first is chaotic, and considered a member of a Gaussian orthogonal ensemble (GOE) of random matrices. The second is integrable, having either equally spaced levels or levels with spacings satisfying a Poisson distribution. The resulting nearest-neighbor-spacing (NNS) distribution of the energy levels of the system is nearly Poissonian in both cases when the analysis involves a large number of levels. If a limited number of levels is considered in each case, deviations from the Poisson distribution are observed. When the regular part of the Hamiltonian is an oscillator with a limited number of phonons, the resulting NNS distribution can be considered as a superposition of independent sequences of levels with GOE statistics when the oscillator energy quantum is larger than the mean spacing of the other term of the Hamiltonian. This distribution has a shape intermediate between the Wigner and the Poisson, and gradually approaches the latter when the number of phonons is increased. This transitional behavior is well reproduced by averaging the level-repulsion function, which may be considered as a justification for a method, recently suggested to calculate the NNS distributions for systems with mixed classical dynamics. [S1063-651X(96)12409-0]

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I. INTRODUCTION

Random matrix theories have been successfully used to describe a wide variety of chaotic systems [1]. For a bound quantal system with time-reversal symmetry (or, equivalently, antiunitary symmetry), energy levels are determined by diagonalizing a Hamiltonian matrix which is real and symmetric. When the classical counterpart of such a system is chaotic, the Hamiltonian matrix can be regarded as a typical member of the Gaussian orthogonal ensemble (GOE) of random real symmetric matrices for which all the matrix elements (diagonal and off-diagonal) have the same Gaussian distribution. The exact expression for the nearest-neighbor-spacing (NNS) distribution of levels of the GOE is quite cumbersome. This distribution can, however, be closely approximated by a simple function, known as the Wigner distribution,

$$P_{\mathrm{Wigner}}(s)=(\pi s/2)\exp(-\pi s^2/4). \tag{1}$$

When the classical counterpart of the system is regular, the NNS distribution is, in general, well represented by a Poisson distribution:

$$P_{\mathrm{Poisson}}(s)=\exp(-s). \tag{2}$$

Among the exceptional cases is the multidimensional harmonic oscillator of incommensurate frequencies, where the NNS distribution is sharply peaked at a finite spacing whose value depends on the frequency ratios [2]. The intermediate regime between regularity and chaos is still the subject of many recent investigations. Various formulas have been proposed to analyze the NNS distribution of levels of mixed systems [3–12]. They depend on one parameter (or more), which can be tuned to interpolate between the Wigner and Poisson distributions. These formulas give different descriptions for the level statistics of mixed system, each providing a suitable description of at least one numerical experiment with a model Hamiltonian. The conclusion that follows from these studies is that, while the level statistics of chaotic and regular bound quantal systems (with the exception of harmonic oscillators) have universal behavior, mixed systems unfortunately do not follow the same rules. Nothing is strange about this. Studies of various classical mixed systems have shown that the transition from regularity to chaos occurs in one of several routes, e.g., a sequence of bifurcations, period doubling, or intermittent transitions [13].

In this paper we consider a special class of mixed systems, in which the degrees of freedom can be divided into two noninteracting groups, one having chaotic dynamics and one regular. The Hamiltonian of such a system is given as a sum of two terms, so that each of the eigenvalues of the total Hamiltonian is expressed as a superposition of two eigenvalues corresponding to the two Hamiltonian terms. Hamiltonians of this form have been successfully used in several models of molecular, nuclear, and solid-state physics. In Sec. II, we generate an energy spectrum corresponding to each of the two terms by treating the level spacing as random numbers having Wigner or Poisson probability densities. We also consider the case when the regular part of the Hamiltonian is modeled as a harmonic oscillator. We calculate the spacing distributions of the levels of the total Hamiltonian for different values of the ratio of level densities of the partial Hamiltonians. The resulting NNS distributions are nearly of a Poissonian shape as far as a large number of levels belonging to each Hamiltonian term are involved. Section III considers the case when the harmonic oscillator is allowed to have a limited number of eigenvalues, and we find that the NNS distribution is approximately well described as that of a mixture of GOE spectra. Section IV applies the recently suggested representation of the level-repulsion function for a mixed system as an average of the corresponding functions for the Poisson and Wigner distributions [12] to derive the NNS of a superposition of GOE spectra, and compares the resulting expression with the distributions obtained in Sec. III. The results of the present paper are summarized in Sec. V.

II. NNS DISTRIBUTION OF LEVELS OF THE MIXED SYSTEM

We consider a system of many degrees of freedom, some of them chaotic and some regular. The Hamiltonian of the system can be decomposed into two terms,

$$H = H_{\text{chaotic}} + H_{\text{regular}}, \tag{3}$$

each corresponding to one of the two groups of degrees of freedom. In the general case, the Hamiltonian has a third term describing the interaction between the two groups. This term is assumed not to influence the nature of dynamics of each part of the Hamiltonian in the class under consideration, and can thus be neglected as far as one is interested only in the fluctuation properties of the spectra. The eigenvalues of H are then expressed by

$$E_{\alpha\beta} = E_\alpha^{\text{chaotic}} + E_\beta^{\text{regular}}, \tag{4}$$

where $E_\alpha^{\text{chaotic}}$ and E_β^{regular} are the eigenvalues of H_{chaotic} and H_{regular}, respectively. We now assume that the NNS distribution of the eigenvalues $E_\alpha^{\text{chaotic}}$ is given by a Wigner distribution (1) with level density $\rho_W(=1)$, while that of the eigenvalues E_β^{regular} is a Poisson distribution with level density ρ_P. We obtain an eigenvalue spectrum for each of H_{chaotic} and H_{regular} by starting from an eigenvalue zero and successively adding spacings generated as random numbers having, respectively, a Wigner or Poisson probability density function, until we reach an energy of 60 (in units of ρ_W^{-1}). Then we compose a spectrum for the total Hamiltonian H by adding the eigenvalues of the partial Hamiltonians according to Eq. (4) and rearranging the resulting values of $E_{\alpha\beta} \leqslant 60$ in an increasing order. The energy spectrum obtained in this way will not be uniform. Indeed, the number of eigenvalues with energy less than E will then approximately be given by

$$\begin{aligned} N(E) &= \int_0^E dE_1 \int_0^E dE_2 N^{\text{chaotic}}(E_1) N^{\text{regular}}(E_2) \\ &\quad \times \delta(E - E_1 - E_2) \\ &= \rho_W \rho_P E^2/2. \end{aligned} \tag{5}$$

The corresponding level density is

$$\rho = \frac{dN(E)}{dE} = \rho_W \rho_P E. \tag{6}$$

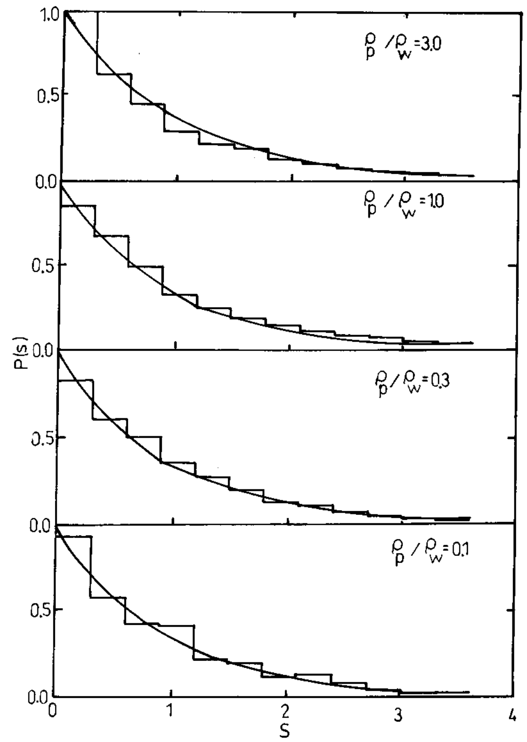

FIG. 1. NNS distributions of the energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner spacing distribution, and one having a Poissonian spectrum, for different values of the level-density ratios of the partial spectra. The smooth curves show the Poisson distribution.

Renormalizing the resulting spectrum by multiplying each NNS by the value of ρ at the midvalue of energies of the neighboring levels, we obtain the required NNS distribution of eigenvalues of the Hamiltonian H. The results of calculation are shown in Fig. 1 for four values of $\rho_P/\rho_W = 0.1$, 0.3, 1, and 3, where the distributions involve 359, 890, 2170, and 3268 spacings, respectively. We see from the figure that the obtained NNS distributions are all very close to a Poisson distribution.

We repeated the same procedure for the case when the regular part of the Hamiltonian describes a harmonic oscillator of frequency ω with energy levels given (to within an additive constant depending on the number of regular degrees of freedom) by

$$E_\beta^{\text{regular}} = \hbar\omega\beta, \quad \beta = 0,1,2, \ldots \tag{7}$$

and level spacing $\rho_0 = 1/\hbar\omega$. The resulting NNS distributions for level-density ratios $\rho_0/\rho_W = 0.1$, 0.3, 1, and 3 are shown in Fig. 2. Again we find that the resulting distributions are nearly Poissonians.

The result of the comparisons shown in Figs. 1 and 2 can be easily understood if we note that the levels of the Hamiltonian under consideration (in which the regular and chaotic coordinates are separable) are in fact a combination of a large number ($\approx 60\rho_P$ or $60/\hbar\omega$) of independent sequences of GOE spectra, each sequence corresponding to a fixed value of E_β^{regular} which is expected [1] to have a Poissonian NNS distribution. This point will be discussed in more detail

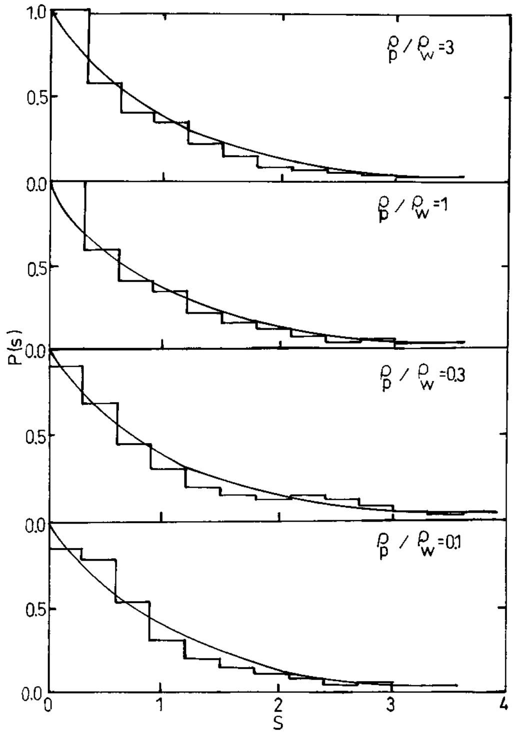

FIG. 2. NNS distributions of the energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner spacing distribution, and one representing a harmonic oscillator, for different values of the level-density ratios of the partial spectra. The smooth curves show the Poisson distribution.

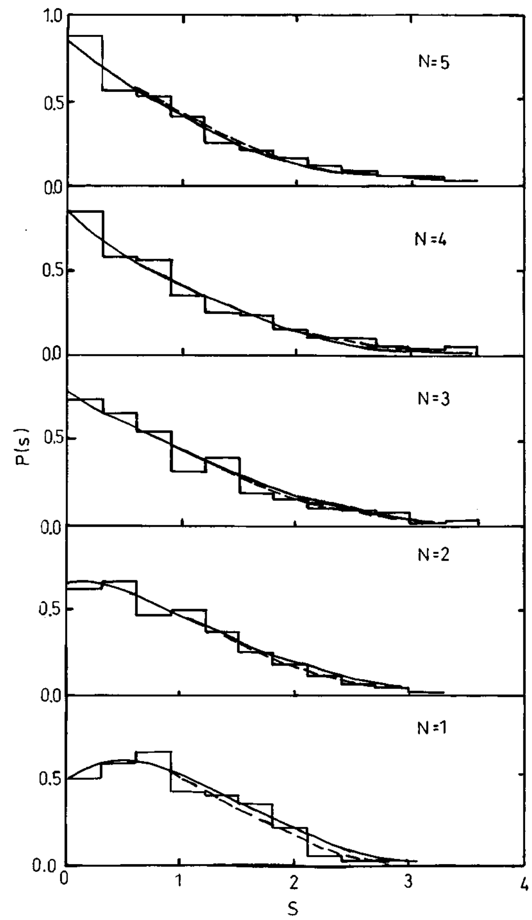

FIG. 3. NNS distributions of energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner spacing distribution, and one representing a harmonic oscillator having at most N phonons, for level-density ratios of the partial spectra $\rho_0/\rho_W=0.1$. The smooth curves show the NNS distributions for a random superposition of N independent GOE spectra. The dotted curves are the distributions calculated using the assumption that the level-repulsion function is given by (20) as an average of the corresponding functions for the Poisson and Wigner distributions.

in Sec. III by considering the case when the regular part of the Hamiltonian is a harmonic oscillator with limited number of eigenvalues.

III. SUPERPOSITION OF SEQUENCES OF GOE SPECTRA

We shall now consider the case when the regular part of the Hamiltonian (3) has a finite number of eigenvalues. In particular we take this part of the Hamiltonian as that of a harmonic oscillator with energy levels

$$E_\beta^{\text{regular}}=\hbar\omega\beta, \quad \beta=0,1,2,\ldots,N. \tag{8}$$

The results to be obtained here will approach the calculation presented in Sec. II when we take the limit of large $N(\approx 60/\hbar\omega)$. We note that allowing only small values of N is not very unrealistic. In nuclear physics, for example, many successful models are based on representing the nuclear Hamiltonian as a sum of intrinsic (interacting particles or quasiparticles) and collective terms as well as a term standing for the interaction between single-particle and collective degrees of freedom [14]. Shell model calculations show that the residual nucleon-nucleon interaction makes the single-particle motion chaotic [15]. On the other hand, the collective part of the Hamiltonian often has a simple integrable form. In particular, when the collective motion is mainly vibrational, it is usually modeled by a harmonic oscillator. The coexistence of regular collective nuclear dynamics with the intrinsic chaoticity has indeed been illustrated in many recent investigations (e.g., [16]). Thus, if we neglect the interaction between single-particle and collective degrees of freedom, we can write the nuclear Hamiltonian in the form given by Eq. (3). Soloviev has recently shown [17] that the role of this interaction increases with increasing excitation energy, and thus the number of oscillator quanta (phonons). Thus the clear separation of intrinsic and collective degrees of freedom which leads to a Hamiltonian of form (3) is possible only when the number N of oscillator quanta is small.

Figures 3–6 show the histograms of the NNS distributions for the levels of the Hamiltonian (3) calculated according to the procedure described in Sec. II, when the spacings of $E_\alpha^{\text{chaotic}}$ have a Wigner distribution, while the values of E_β^{regular} are given by (8) with $N=2$, 3, 4, and 5. We shall now show that, as far as $\hbar\omega$ is larger than the mean level spacing of the chaotic Hamiltonian, the resulting NNS distributions for the levels of the total Hamiltonian can be represented as a superposition of n $(=N+1)$ independent sequences of GOE spectra.

Berry and Robnik [4] and Mehta [1] calculated the NNS

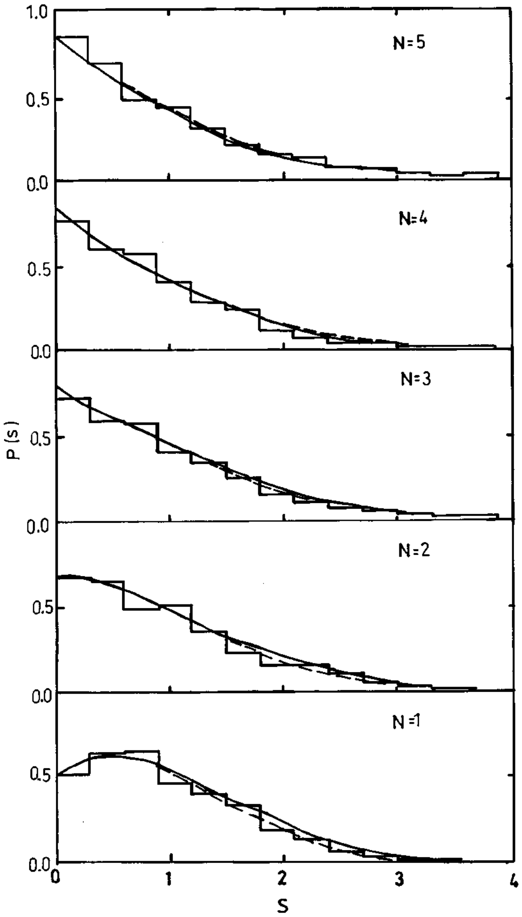

FIG. 4. NNS distributions of energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner spacing distribution, and one representing a harmonic oscillator having at most N phonons, for level-density ratios of the partial spectra $\rho_0/\rho_W=0.3$. The smooth curves show the NNS distributions for a random superposition of N independent GOE spectra. The dotted curves are the distributions calculated using the assumption that the level-repulsion function is given by (20) as an average of the corresponding functions for the Poisson and Wigner distributions.

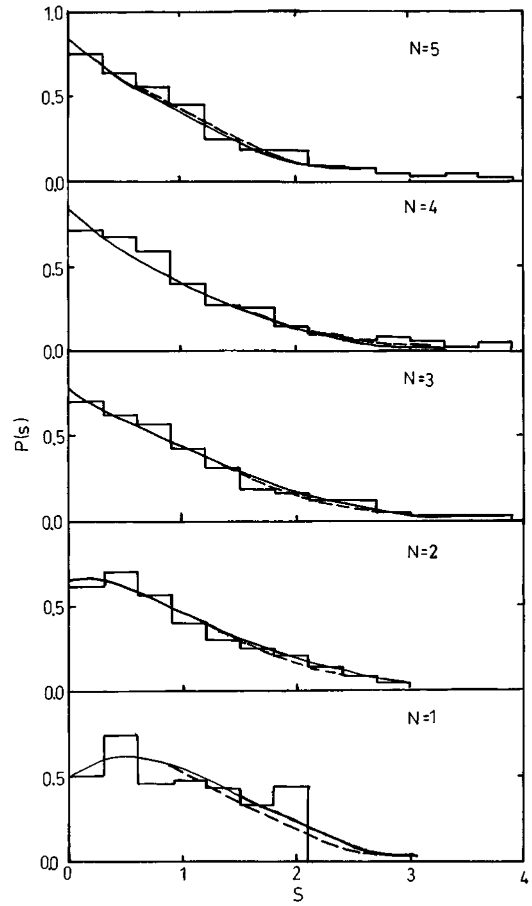

FIG. 5. NNS distributions of energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner spacing distribution, and one representing a harmonic oscillator having at most N phonons, for level-density ratios of the partial spectra $\rho_0/\rho_W=1$. The smooth curves show that NNS distributions for a random superposition of N independent GOE spectra. The dotted curves are the distributions calculated using the assumption that the level-repulsion function is given by (20) as an average of the corresponding functions for the Poisson and Wigner distributions.

distribution of a mixed sequence, resulting from a random superposition of n uncorrelated sequences of energy levels. According to these authors, if the level density of the ith sequence is ρ_i, and if the NNS distribution of levels of this sequence is $P_i(x_I)$ where $x_i=f_is$ and $f_i=\rho_i/\Sigma\rho_i$, the cumulative spacing distribution is

$$W_i(x_i)=\int_0^{x_i}P_i(x)dx, \tag{9}$$

and the probability that, in a given interval of length x_i, there is no level belonging to the ith sequence is

$$E_i(x_i)=\int_{x_i}^{\infty}[1-W_i(x)]dx. \tag{10}$$

Then the probability that a given interval of length s does not contain any of the levels of the mixed sequence is given by

$$E(s)=\prod_i E_i(f_is). \tag{11}$$

The NNS distribution of the mixed system can be obtained by differentiating (11) twice, which yields

$$P(s)=E(s)\left\{\sum_i f_i^2\frac{P_i(f_is)}{E_i(f_is)}+\left[\sum_i f_i\frac{1-W_i(f_is)}{E_i(f_is)}\right]^2 - \sum_i\left[f_i\frac{1-W_i(f_is)}{E_i(f_is)}\right]^2\right\}. \tag{12}$$

If all the n individual sequences have the same level densities, so that $f_i=1/n$, and if the NNS distribution of the levels of each is a Wigner distribution, then (12) becomes

$$P(s)=\frac{1}{n}\left[\operatorname{erfc}\left(\frac{s\sqrt{\pi}}{2N}\right)\right]^n Q(s)\left[\frac{\pi s}{2n}+(n-1)Q(s)\right], \tag{13}$$

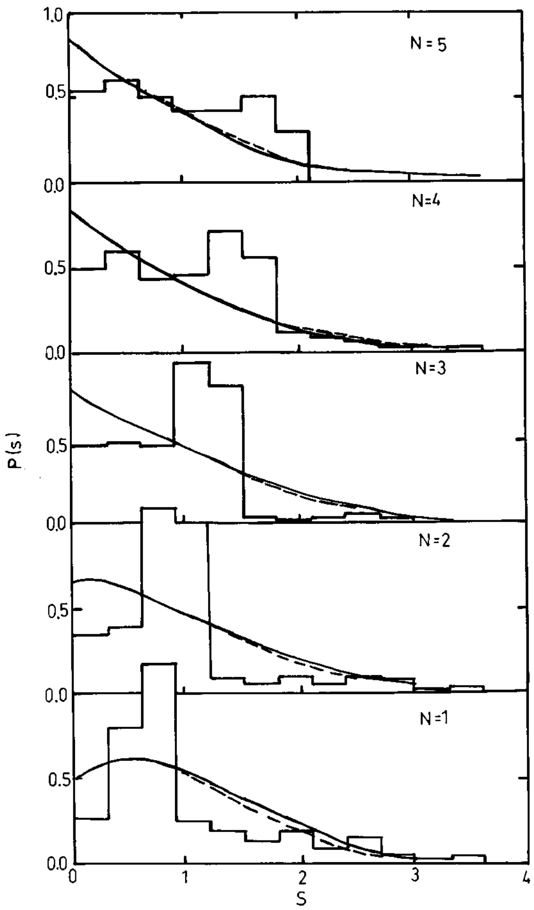

FIG. 6. NNS distributions of energy levels of a Hamiltonian having two terms, one with a spectrum having a Wigner distribution, and one representing a harmonic oscillator having at most N phonons, for level-density ratios of the partial spectra $\rho_0/\rho_W=3$. The smooth curves show the NNS distributions for a random superposition of N independent GOE spectra. The dotted curves are the distributions calculated using the assumption that the level-repulsion function is given by (20) as an average of the corresponding functions for the Poisson and Wigner distributions.

where $Q(s)=e^{-\pi s^2/4n^2}/\mathrm{erfc}(s\sqrt{\pi}/2n)$ and $\mathrm{erfc}(x)$ is the complementary error function. It is interesting to note that, according to (13),

$$P(0)=1-\frac{1}{n} \tag{14}$$

does not vanish except in the case of $n=1$. It slowly approaches unity as n tends to infinity, showing the gradual transition of the NNS distribution of the mixed sequence toward the Poisson distribution as the number of its constituting sequences increases.

We have used (13) to calculate the NNS distributions of levels of $n=N+1$ independent sequences of GOE spectra. The results of calculation are shown as smooth curves in Figs. 3–6 and compared with the corresponding NNS distributions of the Hamiltonian (3) obtained above for the case in which $E_\beta^{\mathrm{regular}}$ is given by (8). The agreement between the curves and histograms is very good, except in the cases in which $\hbar\omega$ is smaller than the mean spacing of eigenvalues of the chaotic term of the Hamiltonian. In the latter cases, many of the nearest neighbors have the same values of $E_\alpha^{\mathrm{chaotic}}$, thus contributing to the NNS distributions with nearly equal spacings.

IV. AVERAGING THE LEVEL-REPULSION FUNCTION

Recently [12], a method was suggested to evaluate the NNS distributions of levels of systems with mixed regular and chaotic dynamics. The starting point of this method is the well-known expression of the NNS distribution $P(s)$ in terms of the level-repulsion function $r(s)$:

$$P(s)=r(s)\exp\left[-\int_0^s r(x)dx\right], \tag{15}$$

which has been derived using simple probability arguments [15]. The level-repulsion function is defined so that $r(s)ds$ is the conditional probability that, given a level at energy E, there is one level in the interval ds provided that there are no levels in the interval $(E,E+s)$. The Poisson distribution (2) can be obtained from (15) by taking

$$r_{\mathrm{Poisson}}(s)=1, \tag{16}$$

which is consistent with the fact that, in the regular regime, the conditional probability density of finding a level in a given spacing interval does not depend on the length of this interval. On the other hand, the Wigner formula (1) for the NNS distribution of levels with GOE statistics is obtained by the following choice of the level-repulsion function:

$$r_{\mathrm{Wigner}}(s)=\tfrac{1}{2}\pi s, \tag{17}$$

where the constant factor ensures a unit average level spacing. Mixed systems have NNS distributions intermediate between the Poisson and Wigner distributions. The celebrated Brody's formula for the level spacing of mixed systems is obtained by assuming a fractional power dependence of the level-repulsion function [3], $r(s)\propto s^\beta$, which smoothly interpolates between the Poisson ($\beta=0$) and Wigner ($\beta=1$) distributions through the parameter β which, unfortunately, cannot be explicitly related to the dynamics of the system. In Ref. [12], it has been assumed that the level-repulsion function for the mixed system can be obtained by averaging the corresponding functions for the regular and chaotic regimes with weights given by the fractional phase-space volumes of their classical motion:

$$r_{\mathrm{mixed}}(s)=q\,r_{\mathrm{Poisson}}(s)+(1-q)r_{\mathrm{Wigner}}(s), \tag{18}$$

where q is the fractional volume of the regular domain of the classical phase space. Equation (18) is obtained by applying Berry's parameter-space method [18], in which the NNS distribution and thus the level-repulsion function $r(s)$ are obtained as an ensemble average of a δ function, to the case when the Hamiltonian ensemble is divided into two subensembles, one for the regular motion and one for the chaotic. Substituting (18) into (15) yields

$$P_{\mathrm{mixed}}(s)=[q+\tfrac{1}{2}\pi(1-q)s]\exp[-qs-\tfrac{1}{4}\pi(1-q)s^2]. \tag{19}$$

This formula was tested in [12] by an analysis of the NNS distributions of energy levels of a hydrogen atom in a uniform magnetic field [8]. The extracted values of the parameter q for all strengths of the magnetic field considered in [8] were found consistent with the corresponding values obtained in the classical-mechanical analysis. Equation (19) was also used in [19] to provide a reasonable description of the level spacing distribution of low-lying excited states of a large number of atomic nuclei, and to obtain a rapid estimate for their fractional phase-space volumes.

The purpose of this section is to make use of the calculation reported in Sec. III for the case when the regular part of the Hamiltonian is modeled as a harmonic oscillator allowed to have only n eigenvalues, to provide a further justification of the averaging of the level-repulsion function introduced by Eq. (18). To do this, let us consider a given label of the Hamiltonian (3) with energy, say, $E_{\alpha\beta}$ given by Eq. (4). The next level will either belong to the same eigenstate of energy $\beta\hbar\omega$ of the partial Hamiltonian H_{regular} or to one of the $n-1$ eigenstates with different eigenvalues $\beta'\hbar\omega$. The former case will occur with a probability of $1/n$ and, in this case, the distance to the second level will be drawn from a Wigner distribution. The latter case will have a probability of $1-1/n$, and the probability density of the corresponding level spacing will be given by the Poisson law because the eigenvalues of both parts of the Hamiltonian are uncorrelated. Therefore, an approximate description of the NNS distribution may be obtained by assuming that the probability density that the next level occurs at a distance s is given by

$$r(s)=(1-1/n)r_{\text{Poisson}}(s)+(1/n)r_{\text{Wigner}}(s). \tag{20}$$

Comparing (18) and (20), we see that this method of averaging the level-repulsion function will lead to a NNS distribution given by Eq. (19), with the parameter q given by

$$q=1-1/n. \tag{21}$$

We first note that the resulting distribution has the correct value (14) at $s=0$. We now compare this distribution with the exact distribution (13) for a mixture of n independent GOE spectra. The result of the comparison are shown in Figs. 3–6 side by side with the histograms representing the results of the numerical calculations of Sec. III. As the figures show, the agreement between the exact and approximate expressions is reasonable, at least within the accuracy required for the analysis of the histograms that are conventionally used to describe the level-spacing distributions.

V. SUMMARY AND CONCLUSION

In several models successfully applied in molecular, nuclear, and solid-state physics, the Hamiltonian of the system under consideration is written as a sum of two terms, describing the intrinsic and collective motion, respectively. Often, the intrinsic degrees of freedom have classically chaotic dynamics whereas the collective degrees of freedom are assumed to be regular. In this case the system is described by a Hamiltonian of the form (3). We constructed a possible eigenvalue spectrum for the chaotic part of such a Hamiltonian by sequentially increasing the eigenvalues by random spacings generated by a Wigner distribution. The eigenvalues of the regular term are obtained by generating random spacings according to a Poisson distribution. The case when the regular part of the Hamiltonian is modeled by a harmonic oscillator was also considered. As far as a sufficient number of eigenvalues of each term of the Hamiltonian are involved in constructing the spectrum, the resulting spacing distribution is Poissonian (Figs. 1 and 2).

We then considered the case when the regular part of the Hamiltonian is that of a harmonic oscillator with eigenvalues restricted to a finite number of oscillator quanta. We show that the NNS distribution of levels of the total Hamiltonian can be reproduced by assuming that the spectrum can be represented as a random superposition of sequences of independent spectra, each described by a GOE. The spacing distributions fall between the Wigner and Poisson distributions, and become closer to the latter as the involved number of levels of the regular Hamiltonian increases (Figs. 3–5). An exception is the case when the regular term of the Hamiltonian models a harmonic oscillator with an eigenvalue mean spacing smaller than that of the chaotic Hamiltonian (Fig. 6). In this case, when the spectrum involves a small number of oscillator eigenvalues, the NNS distribution of the total spectrum resembles that of a multidimensional oscillator of incommensurable frequencies [2]. As the number of involved oscillator quanta increases, we again observe a transition of the NNS distribution to the Poissonian shape.

Finally, we examined the validity of the representation of the level-repulsion function as an average of the corresponding functions for the Wigner and Poisson distributions [Eq. (18)], which was recently suggested as a basis for obtaining the NNS distribution for mixed systems [12]. We used this assumption in Eq. (20) to calculate an approximate expression for the NNS distribution of eigenvalues of the Hamiltonian (3) when the regular part is that of a harmonic oscillator with a limited number of eigenvalues whose exact expression is given by Eq. (13). Figures 3–6 show that the exact and approximate distributions agree reasonably well, which may be considered a justification for the application of Eq. (19) to the analysis of the NNS distributions of mixed systems, at least in cases when the Hamiltonian can be approximated in the form given by Eq. (3).

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